PG Sem-2 Notes CC6 Unit -V

BINOMIAL PROBABILITY DISTRIBUTION

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What Is Binomial Distribution?

The binomial distribution is a probability distribution that summarizes the likelihood that a value will take one of two independent values under a given set of parameters or assumptions. The underlying assumptions of the binomial distribution are that there is only one outcome for each trial, that each trial has the same probability of success, and that each trial is mutually exclusive, or independent of each other.

Understanding Binomial Distribution

The binomial distribution is a common discrete distribution used in statistics, as opposed to a continuous distribution, such as the normal distribution. This is because the binomial distribution only counts two states, typically represented as 1 (for a success) or 0 (for a failure) given a number of trials in the data. The binomial distribution, therefore, represents the probability for x successes in n trials, given a success probability p for each trial.

Binomial distribution summarizes the number of trials, or observations when each trial has the same probability of attaining one particular value. The binomial distribution determines the probability of observing a specified number of successful outcomes in a specified number of trials.

The binomial distribution is often used in social science statistics as a building block for models for dichotomous outcome variables, like whether a Republican or Democrat will win an upcoming election or whether an individual will die within a specified period of time, etc.

Binomial Experiment

A binomial experiment is a statistical experiment that has the following properties:

The experiment consists of n repeated trials.

Each trial can result in just two possible outcomes. We call one of these outcomes a success and the other, a failure.

The probability of success, denoted by P, is the same on every trial.

The trials are independent; that is, the outcome on one trial does not affect the outcome on other trials.

Consider the following statistical experiment. You flip a coin 2 times and count the number of times the coin lands on heads. This is a binomial experiment because:

The experiment consists of repeated trials. We flip a coin 2 times.

Each trial can result in just two possible outcomes - heads or tails.

The probability of success is constant - 0.5 on every trial.

The trials are independent; that is, getting heads on one trial does not affect whether we get heads on other trials.

Notation

The following notation is helpful, when we talk about binomial probability.

x: The number of successes that result from the binomial experiment.

n: The number of trials in the binomial experiment.

P: The probability of success on an individual trial.

Q: The probability of failure on an individual trial. (This is equal to 1 - P.)

n!: The factorial of n (also known as n factorial).

b(x; n, P): Binomial probability - the probability that an n-trial binomial experiment results in exactly x successes, when the probability of success on an individual trial is P.

nCr: The number of combinations of n things, taken r at a time.

Binomial Distribution

A binomial random variable is the number of successes x in n repeated trials of a binomial experiment. The probability distribution of a binomial random variable is called a binomial distribution.

Binomial Formula and Binomial Probability

The binomial probability refers to the probability that a binomial experiment results in exactly x successes. For example, in the above table, we see that the binomial probability of getting exactly one head in two coin flips is 0.50.

Given x, n, and P, we can compute the binomial probability based on the binomial formula:

Binomial Formula. Suppose a binomial experiment consists of n trials and results in x successes. If the probability of success on an individual trial is P, then the binomial probability is:

b(x; n, P) = nCx * Px * (1 - P)n - x

or

b(x; n, P) = { n! / [x! (n - x)!] } * Px * (1 - P)n – x

Example 1

Suppose a die is tossed 5 times. What is the probability of getting exactly 2 fours?

Solution: This is a binomial experiment in which the number of trials is equal to 5, the number of successes is equal to 2, and the probability of success on a single trial is 1/6 or about 0.167. Therefore, the binomial probability is:

b(2; 5, 0.167) = 5C2 * (0.167)^2 * (0.833)^3

b(2; 5, 0.167) = 0.161

Example 2 :

The lifetime risk of developing pancreatic cancer is about one in 78 (1.28%). Suppose we randomly sample 200 people. Let X= the number of people who will develop pancreatic cancer.

a. What is the probability distribution for *X*?
b. Using the formulas, calculate the (i) mean and (ii) standard deviation of *X*.
c. Use your calculator to find the probability that at most eight people develop pancreatic cancer.
d. Is it more likely that five or six people will develop pancreatic cancer? Justify your answer numerically.

Solution

a. X∼B(200,0.0128)X∼B(200,0.0128)
    a. Mean=np=200(0.0128)=2.56=np=200(0.0128)=2.56
    b. Standard Deviation=√npq=√(200)(0.0128)(0.9872)≈1.5897Standard Deviation=npq=(200)(0.0128)(0.9872)≈1.5897
b. Using the TI-83, 83+, 84 calculator: P(x≤8)=binomcdf(200,0.0128,8)=0.9988P(x≤8)=binomcdf(200,0.0128,8)=0.9988
c. P(x=5)=binompdf(200,0.0128,5)=0.0707P(x=5)=binompdf(200,0.0128,5)=0.0707 P(x=6)=binompdf(200,0.0128,6)=0.0298P(x=6)=binompdf(200,0.0128,6)=0.0298 So P(x=5)>P(x=6) P(x=5)>P(x=6); it is more likely that five people will develop cancer than six.

References :

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